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An inverse problem for retrieving time dependency of heat flux in metal cutting via linear programming

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Abstract

The paper addresses the inverse heat transfer problem, in which the dependence of the flux on time is determined on the basis of the measured temperatures in remote points of the tool body. The nonlinear heat equation is considered since the properties of the tool material are temperature-dependent. The machining time interval is divided into subintervals within which the change in the tool material properties is ignored. Therefore, it is possible to consider a sequence of linear programming problems, each of which minimizes the maximum deviation of the measured temperatures from the calculated ones.

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Keywords: Tool temperature; inverse heat transfer problem; heat flux

1. Introduction

Thermal phenomena in metal cutting are one of the keys to the analysis and understanding of the machining processes since they strongly affect to production rate and quality of the products manufactured. The use of the boundary-value problem for the heat equation, so-called direct problem, is possible only if all its numerical parameters are known [1]. One of the most important of them in the case of machining is the heat flux through the tool-chip

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interface. Experimental methods for measuring the flux are either absent or extremely complex. At the same time, there are relatively accessible methods for determining the temperature in the tool, for instance, thermocouples. They ensure indirect information for heat flux retrieval. In other words, the so-called inverse heat conduction problems are an alternative to direct measurement [2].

However, the most negative feature of the inverse heat conduction problems is their ill-posedness [1]. In the computational sense, it means that inevitable errors in the measured data lead to a critical distortion of the sought solution of the inverse problem. For instance, temperature error of a few percent can lead to critically amplified oscillations in the retrieved flux. The basic idea to overcome such ill-posedness is to provide an additional information about the behavior of the solution [3]. This leads to a lack of universal methods and commercial software that can solve the inverse problems or tasks in question. The most popular approach to retrieve the heat flux flowing into a cutting tool in machining is based on sequential function specification method [4-7]. The main limitation of this method is the fact that it requires careful tuning for each individual case. Even after careful tuning the method provides reliable results only for the average flux value, but does not resolve its time dependency correctly. Other, more exotic methods, such as an adapted method of steepest descent [8, 9] or a method that is based on a recursive function transfer form [10] work with either non-temperature-dependent thermal properties [8, 9] or consider imitational [10], instead of actual, machining processes.

The main idea of this study is to develop and validate a new method for the retrieval of a heat flux into a tool on the basis of the measured temperatures via linear programming. The advantage of this technique is that it resolves the time dependency of the flux into the tool.

Nomenclature

u, u_{ext}	Temperature and temperature of the environment, °C.
ρ	Density, (kg/m ³).
c_p	Specific heat, J/(kg K).
k	Thermal conductivity, W/(m K).
TC#	Thermocouple number #.
$q(t)$	Heat flux, W/mm ² .
h	Coefficient of heat exchange with the environment, W/(mm ² K).
W, Q	Tool-chip contact surface and surface of the tool exposed to the environment, respectively.
T	Experiment and modelling time, s.

2. Experimental setup

The proposed approach is based on measured temperature readings from thermocouples installed in a cutting tool. As mentioned above, the inverse heat transfer problem is ill-posed, hence it is sensitive both to measurement errors and to the shortcomings of the numerical model of the cutting tool. Therefore, a proper design of the experimental setup is necessary to minimize both potential error contributions.

The following design requirements and solutions are introduced in the current study: (1) a solid body of the cutting tool to avoid thermal contacts in the tool assembly among anvil, cutting insert, clamp plate, etc.; (2) the workpiece and tool material pair and cutting conditions must ensure the absence of tool wear and consequently an additional heat source with unknown characteristics; (3) a simple geometry of the tool body must enable adequate numerical modelling via 2D model; (4) the locations of the thermocouples must uniformly cover the range of measured temperatures to enable selection temperature datasets that are not distorted by temperature gradient, but are still sensitive enough to the phenomenon being investigated.

That is why, the experimental setup is based on orthogonal cutting test. The tool is a solid rectangular bar with 25 mm square cross-section and 157 mm length with clearance angle $\alpha = 7$ degrees and flat rake ($\gamma = 0^\circ$). A special collet to mount the tool on a dynamometer ensures simple and easy to model geometry of the thermal contact between the tool itself and the collet (Fig. 1).

The high speed steel Vanadis® 23 SuperClean (Uddeholm) [11] hardened to 64 HRC is the material of the tool. Table 1 reports the thermal properties for the tool that were used in the finite element analysis. The workpiece was 6082 aluminum alloy disk of 470 mm in diameter and 4 mm in thickness.

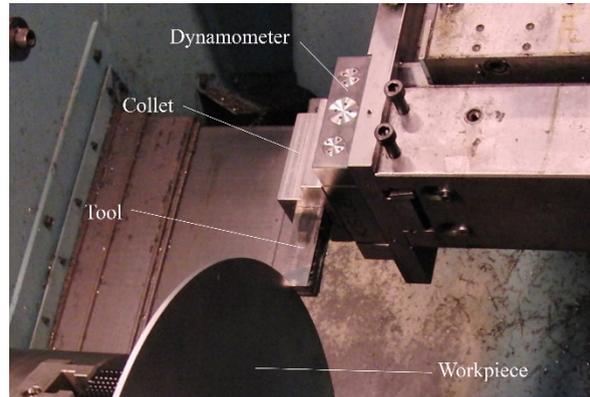


Fig. 1. Photograph of experimental setup.

Table 1. The thermal properties of the high speed steel tool Vanadis® 23 SuperClean (Uddeholm) [10].

Temperature range (°C)	$20 \leq u \leq 400$	$400 \leq u \leq 600$
Density	$-0.28947 u + 7985.8$	$-0.32500 u + 8000$
Thermal conductivity	$0.010526 u + 23.789$	$-0.005 u + 30$
Specific heat	$0.23684 u + 415.26$	$0.45 u + 510$

The simple tool geometry with center-plane symmetry and locations of the thermocouple junctions on the center plane of the tool allows to use 2D model in the FEA (Fig. 2a). A set of numerical simulations at the varying thermocouple positions along the tool body was carried out in order to determine the above mentioned coverage of the range of measured temperatures. The selected positions correspond to the Fibonacci numbers (Fig. 2b). According to recommendations [12], eight SC-KK-K-40-1M fine-gauge Kapton insulated thermocouples (Omega Engineering) with the wire diameter 80 μm were inserted into the deep thin holes of 0.5 mm diameter and 12.5 mm depth filled by a high thermal conductivity silicone paste Omegatherm 201.

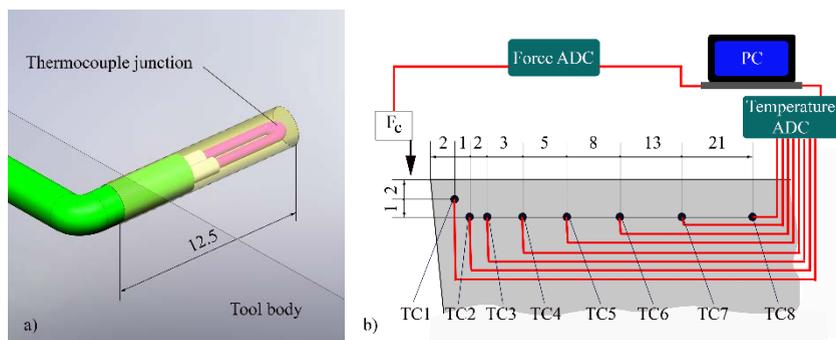


Fig. 2. Scheme of the thermocouple installation (a) and their positions along the tool body (b).

The experimental setup also consists of a dynamometer (Kistler 9129AA) for cutting force registration and high density thermocouple module (NI9213) for thermocouple signal acquisition.

3. Modelling

3.1. The direct heat transfer problem

Because the tool thermal properties are temperature dependents (Table 1), the nonlinear equation (Eq.1) governs the heat balance in any interior points of the tool body. To solve it, the boundary (Eq.2, Eq.3) and initial (Eq.4) conditions are necessary. The boundary is divided into two parts: (1) W , through which a heat flux $q(t)$ is flowing into the tool and Q , which is exposed to the environment (Fig.3). The coefficient h (Eq.3) was taken from [13]. So, as soon as $q(t)$ is determined, the problem Eq.1-Eq.4 can be solved as a direct heat transfer problem.

$$k(u) \frac{\partial^2 u}{\partial x^2} + k(u) \frac{\partial^2 u}{\partial y^2} = \rho c_p(u) \frac{\partial u}{\partial t}, \tag{1}$$

$$k(u) \frac{\partial u}{\partial n} \Big|_{(x,y) \in W} = q(t), \tag{2}$$

$$-k(u) \frac{\partial u}{\partial n} \Big|_{(x,y) \in Q} = h(u - u_{ext}), \tag{3}$$

$$u|_{t=0} = u_{ext}. \tag{4}$$

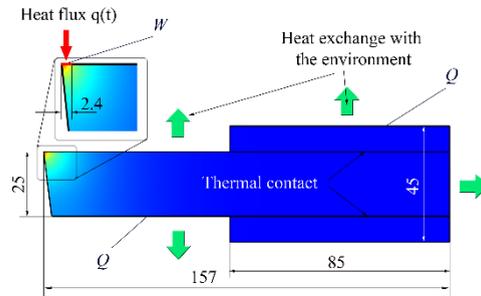


Fig. 3. Boundary value problem.

The determination of the $q(t)$ is based on the optimization technique for reasons of matching the measured and the FE modelled temperatures.

3.2. Optimization technique

The time dependent heat flux $q(t)$ is retrieved in the form of a piecewise constant function over time. For this aim the modelling time T is presented as a discrete set of equally spaced time moments with 1 sec between them. Then, the measured and calculated temperatures are reduced to this discrete set. When reconstructing a piecewise constant function, the modelling time T is divided into stages $s = 1, 2, \dots, S$ with a whole (integer) number of seconds on each stage. Let T_1, T_2, \dots, T_m be the ends of the respective stages.

When considering heat transfer within a stage, we formulate three linear boundary value problems for the heat equation at the stage s : (P1) cooling down problem, (P2) sensitivity problem, (P3) temperature reconstruction problem.

$$k_s \frac{\partial^2 u^{(s,P\#)}}{\partial x^2} + k_s \frac{\partial^2 u^{(s,P\#)}}{\partial y^2} = \rho_s c_p^{(s)} \frac{\partial u^{(s,P\#)}}{\partial t}, \tag{5}$$

$$k_s \frac{\partial u^{(s,P\#)}}{\partial n} \Big|_{(x,y) \in W} = \begin{cases} 0, & \text{for } P1 \\ 1 \text{ W/mm}^2, & \text{for } P2, \\ \mu \text{ W/mm}^2, & \text{for } P3 \end{cases} \quad (6)$$

$$-k_s \frac{\partial u^{(s,P\#)}}{\partial n} \Big|_{(x,y) \in Q} = h(u^{(s,P\#)} - u_{ext}), \quad (7)$$

$$u^{(s,P\#)} \Big|_{t=T_{s-1}} = u^{(s-1,P2)} \Big|_{t=T_{s-1}}, \quad (8)$$

where k_s , ρ_s , $c_p^{(s)}$ are the temperature independent thermal properties on the stage s , and $u^{(s,P\#)}$ is the solution of the problem $P\#$ on the stage s .

One can note that material thermal properties on the stage s do not depend on temperature, but they are the functions of spatial variables. The variable μ is defined with the help of linear programming problem. The point is that due to the linearity, the solution of $P3$ on the stage s can be presented as a product $u^{(s,P2)}\mu$, where μ is the calculated value that minimizes the error between modelled and measured temperatures.

To find μ , the optimization problem is formed:

$$v \rightarrow \min, \quad (9)$$

$$|a_{ij}^{(s)}\mu - b_{ij}^{(s)}| \leq v. \quad (10)$$

where

$$a_{ij}^{(s)} = u_{ij}^{(s,P2)} - u_{ij}^{(s,P1)}, \quad (11)$$

$$b_{ij}^{(s)} = w_{ij}^{(s)} - u_{ij}^{(s,P1)}. \quad (12)$$

Here $u_{ij}^{(s,P\#)}$ is the calculated temperature in the problem $P\#$ on the stage s and in the point of j -th thermocouple (TCj) at i -th time moment. In turn, $w_{ij}^{(s)}$ is the measured temperature registered by j -th thermocouple (TCj) at i -th time moment on the stage s . The variable v is the maximum absolute error between the modelled and the measured temperatures on the stage s .

The solution of the problem (9-10) can be reduced to solving an appropriate linear programming problem:

$$v \rightarrow \min, \quad (13)$$

$$\begin{cases} a_{ij}^{(s)}\mu - v \leq b_{ij}^{(s)} \\ a_{ij}^{(s)}\mu - v \geq b_{ij}^{(s)}. \end{cases} \quad (14)$$

4. Results and discussion

The orthogonal machining experiment was performed on a 70-kW SMT 500 CNC lathe with cutting speed $v_c = 150$ m/min and feed $f = 0.3$ mm/rev. The cutting process lasted for 103 s. The dimension of chip-tool contact W has been measured with the optical microscopy and equaled to 2.4 mm \times 4.6 mm. Dynamometer and thermocouple readings are summarized in Fig. 4.

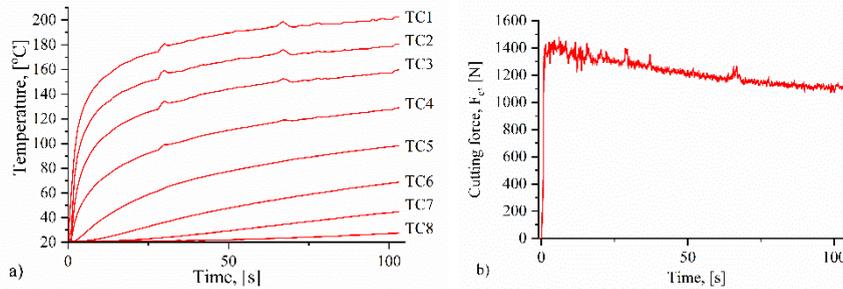


Fig. 4. Measured temperatures $TC_j, j=1, \dots, 8$ (a) and the cutting force F_c (b).

Analysis of the thermocouple readings and test computer experiments showed that TC1 and TC2 are in a zone with an extreme temperature gradient and their readings can significantly deviate from the actual temperatures in the tool body. Therefore, only TC3 – TC8 were used for the heat flux retrieval.

The software solution that implements the strategy described in the section 3 is built around the combination of COMSOL Multiphysics and MATLAB. The implementation strategy is divided into three steps:

Step 0. Generate the initial temperature distribution, which corresponds to the temperature of the environment. The values of the thermal properties correspond to their values at external temperature.

Step 1. Choose the duration of the stage. To do so, the boundary value problem $P2$ (Eq.5)-(Eq.8) is solved. Then, the linear programming problem (Eq.13)-(Eq.14) is generated and the solution ν is found. As mentioned, this value of ν is maximum absolute difference between measured and calculated temperatures on the stage s . The duration of the stage s is a variable and depends on the value ν . Parameter ν is iteratively calculated with one second increment and the duration of stage s corresponds to the least ν found. This mechanism makes the stage duration an adaptive value.

Step 2. Calculate the temperature distribution at the end of the stage. To do this, the boundary problem $P3$ (Eq.5)-(Eq.8) is solved. The solution of the problem $P3$ at the end of the current stage thus becomes the initial distribution for the next stage. It is important to note that on the next stage the new thermal properties corresponding to the temperature distribution are used – thus making the developed solution a non-linear heat transfer case. The algorithm is repeated until the modeling time T is reached.

The retrieved piecewise constant flux is shown in Fig. 5a. Its behavior over time corresponds to the reported one in [13]. To model the temperature distribution over the entire time interval, the oscillations of the heat flux that arise because of the ill-posedness of the inverse heat conduction problem were smoothed. The measured temperatures and the calculated ones are shown in Fig. 5b. As can be seen that a good agreement between the two datasets was obtained, which indicates the viability of the proposed approach. Deviations can be explained by the simplifications adopted in the problem formulation, primarily related to the use of 2D case.

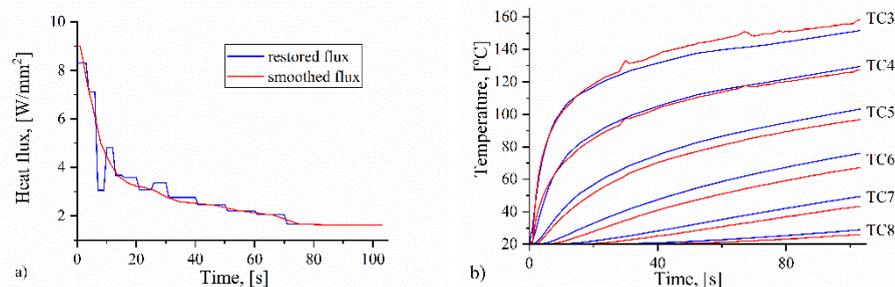


Fig. 5. Retrieved flux (a) and temperatures comparison (b).

In addition, the information obtained makes it possible to estimate quantitatively the fraction of the total cutting power consumed by the tool. First, the total consumed power during cutting equals:

$$P_{total} = Average(F_c) \times v_c = 1259[N] \times 2.5[m/s] = 3145.5[W]. \quad (15)$$

Then, the average specific power consumed by the tool is:

$$p_{tool} = \int_0^{103} q(t)dt / T = 283.29[J/mm^2]/103[s] = 2.75 [W/mm^2]. \quad (16)$$

The measured the area of the tool-chip contact zone is $4.6 \times 2.4 = 11.04 \text{ mm}^2$, thus making the total power consumed by the tool:

$$P_{tool} = 11.04 [mm^2] \times 2.75[W/mm^2] = 30.36[W]. \quad (17)$$

The ratio of the power transferred into the tool to the total process power is 0.97%.

5. Conclusion

The approach based on the linear programming problem was developed and successfully applied to an inverse heat conduction problem when determining the time dependency of the heat flux in orthogonal cutting. The results show that the approach does not require *a priori* information for the functional form of the heat flux behavior and the reliable flux estimation can be obtained both in linear and non-linear heat transfer cases.

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References

- [1] J. V. Beck, B. Blackwell, Ch.R.St. Clair Jr., Inverse Heat Conduction: Ill-Posed Problems, New-York, 1985.
- [2] H. R.B.Orlande, O. Fudym, D. Maillat, R. M.Cotta, Thermal measurements and inverse techniques, CRC Press, Taylor & Francis Group, 2011.
- [3] V. Kryzhanivskyy, V. Bushlya, O. Gutnichenko, I.A. Petrusha, J.-E. Ståhl, Modelling and Experimental Investigation of Cutting Temperature when Rough Turning Hardened Tool Steel with PCBN Tools. Procedia CIRP 2015; 31: 489 – 495.
- [4] V. Norouzifard, M. Hamed, A three-dimensional heat conduction inverse procedure to investigate tool–chip thermal interaction in machining process, Int J Adv Manuf Technol (2014) 74:1637–1648.
- [5] R.F. Brito, S.R. Carvalho, S.M.M. Lima E Silva, Experimental investigation of thermal aspects in a cutting tool using comsol and inverse problem, Applied Thermal Engineering 86 (2015) 60–68.
- [6] F. Samadi, F. Kowsary, A. Sarchami, Estimation of heat flux imposed on the rake face of a cutting tool: A nonlinear, complex geometry inverse heat conduction case study, International Communications in Heat and Mass Transfer 39 (2012) 298–303.
- [7] H.-J. Kim, N.-K. Kim, J.-S. Kwak, Heat flux distribution model by sequential algorithm of inverse heat transfer for determining workpiece temperature in creep feed grinding, International Journal of Machine Tools & Manufacture 46 (2006) 2086–2093.
- [8] C.-H. Huang, L.-C. Jan, R. Li, A. J. Shih, A three-dimensional inverse problem in estimating the applied heat flux of a titanium drilling – Theoretical and experimental studies, International Journal of Heat and Mass Transfer 50 (2007) 3265–3277.
- [9] C.-H. Huang, H.-C. Lo, A three-dimensional inverse problem in predicting the heat fluxes distribution in the cutting tools , Numerical Heat Transfer, Part A, 48: 1009–1034, 2005.
- [10] J.-L. Battaglia, J.-C. Batsale, Estimation of heat flux and temperature in a tool during turning, Inverse Problems in Engineering, 2000, Vol. 8, pp. 435–456.
- [11] http://www.uddeholm.com/files/PB_Uddeholm_vanadis_23_english.pdf.
- [12] M.A. Davies, T. Ueda, R. M'Saoubi, B. Mullany, A. L. Cooke, On the measurement of temperature in material removal processes. CIRP Ann 2007; 56: 581-604.
- [13] V. Kryzhanivskyy, V. Bushlya, O. Gutnichenko, R. M'Saoubi, J.-E. Ståhl, Computational and experimental inverse problem approach for determination of time dependency of heat flux in metal cutting, Procedia CIRP 58 (2017) 122 – 127.